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Heat Transfer Amplification due to Transverse Mode Oscillations

Amichay H. Gross,* Alexander Osherov,† and Alon Gany‡
Technion—Israel Institute of Technology, 32000 Haifa, Israel

The goal of this research is to estimate the magnitude of heat transfer amplification due to combustion instability in the first tangential mode. A theoretical investigation introduced a model consisting of a quasi-steady-state description of the oscillating flow. An experimental facility was constructed, and a few sets of experiments were conducted. The results show that transverse pressure oscillations can increase heat transfer to the walls up to about twofold for pressure oscillations of 50% of the mean pressure. Thus, substantial amplification of heat transfer should be expected in combustors operating under instability conditions. This effect may be particularly crucial in liquid-propellant rocket engines.

Nomenclature

A = amplification/amplitude
c = speed of sound
d = diameter
f = oscillation frequency
h = convection coefficient
k = conduction coefficient
M = Mach number
P = pressure
p = oscillatory pressure component
R = specific gas constant
T = temperature
t = time
U = velocity
v = oscillatory velocity component
x = transverse coordinate
α = thermal diffusivity
γ = specific heat ratio
μ = viscosity
ν = kinematic viscosity
ρ = density

Subscripts

b = bulk flow
in wall = inner wall of test chamber
mid wall = surface of contact
p = pressure oscillation
v = velocity oscillation
water = water flow through heat flux sensor (HFS)
water in = incoming water in HFS
water out = outgoing water in HFS
0 = mean flow

Introduction

PRESSURE and velocity oscillations in the flowfield, associated with combustion instability, may occur under certain operating conditions. Such oscillations can take place in either of three main directions, longitudinal, radial, or tangential[1] (Fig. 1).

Combustion instability can cause augmentation of the heat transfer to the combustor walls. Hence, it can have major implications in liquid-propellant rocket engines. Characterized by high frequencies, the first tangential mode of oscillation has been found to be the most destructive one. Such cases have been reported mainly at the development stages of liquid-propellant rocket engines. Zinn et al.[2] and Dubois and Habiballah[3] revealed that the first tangential mode of combustion instability has the largest amplitude of all transverse oscillation modes. Furthermore, it seems to attain a certain amplitude, which is determined by the form and not the magnitude of excitation. It is also shown that the waveform is approximately sinusoidal. The chamber and nozzle geometries affect the oscillation by determining the acoustic properties of the chamber.

Wall heat transfer amplification due to combustion instability has been investigated by Bogdanoff[4] and Hanby.[5] Those investigations dealt with the first longitudinal mode in which the oscillations have the same direction as the main flow; therefore, for a part of the cycle, the velocity magnitude would become lower, particularily at large amplitudes, causing heat transfer to diminish. When transverse oscillations perpendicular to the main flow are dealt with, the velocity magnitude can only increase, and therefore, heat transfer can only be amplified.

The objective of this research was to predict the amplification of heat transfer to the wall due to transverse oscillations, for various combustor's geometries and operating conditions.

Theoretical Analysis

In the theoretical model developed here, the problem is formulated as transverse oscillations in the flow and their effect on time-averaged heat transfer to the wall, represented by Nusselt number. A computer code was written to solve this problem for a range of conditions, where the pressure oscillation amplitude is the main input (in addition to the gas flow properties), and heat transfer amplification is the output. The model uses the assumptions of a quasi-steady flow and a negligible effect of oscillation frequency on heat transfer amplification. Previous works support these assumptions.[4–10]

Oscillation Wave Amplitude

Because we are dealing with heat transfer to the wall, the region of interest would be near the combustion chamber walls, where combustion rarely takes place. Tangential oscillations at this region can be approximated as transverse oscillations in a rectangular cross section duct of an infinite length (Fig. 2). Results produced using this approach will improve for a larger chamber diameter, as well as for a smaller boundary-layer thickness.

The first step of building the model was deriving the velocity amplitude from the pressure amplitude of the oscillation. The pressure oscillation is considered as our input due to the relative ease of its measurement.

As a basis, we write an approximate (linear) wave motion equation, assuming isentropic flow. These approximations are expected

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*Graduate Student, Faculty of Aerospace Engineering.
†Senior Scientist, Faculty of Aerospace Engineering.
‡Professor, Lena and Ben Fohrman Chair in Aeronautical Engineering, and Head, Fine Rocket Propulsion Center, Faculty of Aerospace Engineering; gany@tx.technion.ac.il. Associate Fellow AIAA.

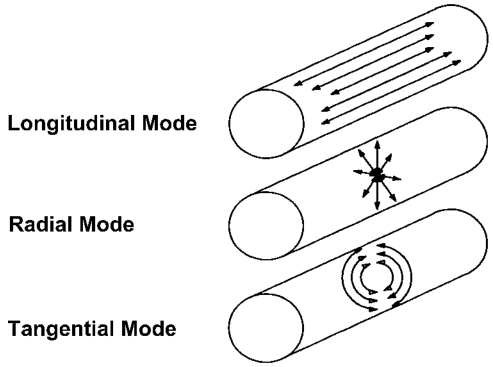

Fig. 1 Different modes of instability.

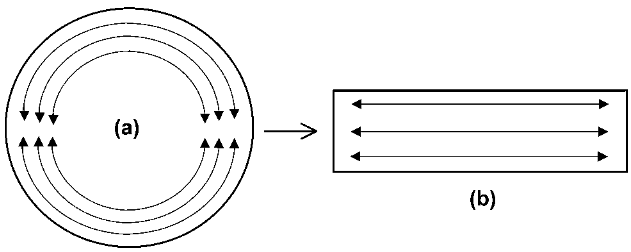

Fig. 2 Oscillation wave motion of a) first tangential mode, cylindrical chamber and b) first transverse mode, rectangular duct.

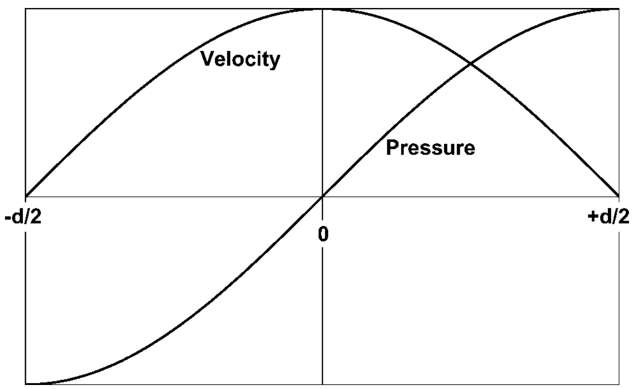

Fig. 3 Pressure and velocity oscillation waveforms.

to result in some inaccuracy of the model, especially for high oscillation frequencies and amplitudes. Quantifying this inaccuracy is difficult, but it may be indicated by experimental data scattering, resulting from nonlinear flow effects and heat losses to the environment.

The pressure and the velocity oscillations have a phase shift of 90 deg, both in time and in location. Therefore, at a certain time, $t = 0$, the pressure amplitude will be minimal and the velocity amplitude will be maximal. The same can be derived for the center, $x = 0$, and the opposite situation for the edges, $x = \pm d/2$ (Fig. 3). The wave equations and their boundary and initial conditions are as follows:

$$c^2 \cdot \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} \tag{1a}$$

where

$$\left.\frac{\partial v}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial v}{\partial t}\right|_{t=0} = 0, \qquad v\left(\frac{\pm d}{2}, t\right) = 0 \tag{1b}$$

$$c^2 \cdot \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{2a}$$

where

$$p(0, t) = 0, \qquad p(x, 0) = 0, \qquad \left.\frac{\partial p}{\partial x}\right|_{x=\pm d/2} = 0 \tag{2b}$$

The solution of the wave equations for the first mode gives the following two waveforms:

$$v(x, t) = A_v \cdot U_0 \cdot \cos[(\pi/d)x] \cdot \cos[(\pi \cdot c/d)t] \tag{3}$$

where $A_v = |v|/U_0$, and

$$p(x, t) = A_p \cdot P_0 \cdot \sin[(\pi/d)x] \cdot \sin[(\pi \cdot c/d)t] \tag{4}$$

where $A_p = |p|/P_0$.

In the case of a circular chamber (Fig. 2a) of a diameter d, the expression for the frequency of the first tangential mode is $S \cdot c/(\pi \cdot d)$, where $S = 1.8413$ (Ref. 1, p. 143).

To correlate the pressure and velocity waves, the conservation of mass and momentum, at a constant cross section, is introduced:

$$\frac{\partial \rho}{\partial t} + v \cdot \frac{\partial \rho}{\partial x} + \rho \cdot \frac{\partial v}{\partial x} = 0 \tag{5}$$

$$\frac{\partial(\rho \cdot v)}{\partial t} + \rho \cdot v \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial(\rho \cdot v)}{\partial x} = -\frac{\partial p}{\partial x} + \mu \cdot \frac{\partial^2 v}{\partial x^2} \tag{6}$$

Uniting the two conservation equations will yield the diffusion equation

$$\rho \cdot \frac{\partial v}{\partial t} + \rho \cdot v \cdot \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial x} + \mu \cdot \frac{\partial^2 v}{\partial x^2} \tag{7}$$

It can be shown that for the waves described in Eqs. (3) and (4) the viscosity term is negligible, and thus, Euler's equation for unsteady flow is obtained:

$$\rho \cdot \frac{\partial v}{\partial t} + \rho \cdot v \cdot \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial x} \tag{8}$$

For the purpose of deriving a correlation between the pressure and velocity oscillation amplitudes, a single point can be considered. For convenience, the center point, in which the velocity gradient is equal to zero, will be taken:

$$\rho \cdot \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}, \qquad \text{at} \qquad x = 0 \tag{9}$$

Applying the wave equations [Eqs. (3) and (4)] will yield

$$\rho \cdot U_0 \cdot A_v \cdot \sin\{[(\pi \cdot c)/d]t\} \cdot [(\pi \cdot c)/d] = P_0 \cdot A_p \cdot \sin\{[(\pi \cdot c)/d]t\} \cdot (\pi/d) \tag{10}$$

For low Mach numbers, density changes can be neglected. When constant density and ideal gas are assumed, the following relation can be arrived at:

$$[c/(R \cdot T_0)] \cdot U_0 \cdot A_v = A_p \tag{11}$$

Applying the common correlation for the speed of sound $c^2 = \gamma \cdot R \cdot T$ and the definition of Mach number $M = U/c$ would result in the correlation between the oscillation's velocity and pressure amplitudes:

$$A_v = A_p/(\gamma \cdot M_0) \tag{12}$$

It can be shown that for isentropic flow the value of M_0 depends only on the area ratio (between the chamber and the choked nozzle cross sections) and the specific heat ratio γ, which has a weak dependency on temperature. Therefore, one may say that for a combustor of a specific geometry, operating at a certain temperature range, the term $\gamma \cdot M_0$ in Eq. (12) will be constant.

For further verification, this correlation was arrived at by Dent,[6] using the Dankwertz–Mickley model for turbulent flows, and by Bogdanoff,[4] using the perturbation approach.

Heat Transfer Amplification

Different models have been considered to correlate between the pressure oscillation amplitude and the heat transfer, represented by Nusselt number, including unsteady boundary-layer models.[11] However, the correlation yielding the most realistic results has been based on a simple model of a quasi-steady flow.

When a negligible dynamic lag of the boundary layer is assumed, the instantaneous (oscillating) boundary layer can be represented by quasi-steady flow characteristics. Therefore, as a basis for the analysis of heat transfer amplification, the known correlation for forced convection in turbulent flows, experimentally proven by Zellnik and Churchill,[12] is taken:

$$Nu \propto Re^{0.8} Pr^{\frac{1}{3}} \tag{13}$$

where Nusselt, Reynolds, and Prandtl numbers *Nu*, *Re*, and *Pr*, respectively, are defined as

$$Nu = (h \cdot d)/k \tag{14a}$$

$$Re = (\rho \cdot |U| \cdot d)/\mu = (|U| \cdot d)/\nu \tag{14b}$$

$$Pr = \mu/(\rho \cdot \alpha) = \nu/\alpha \tag{14c}$$

and $|U|$ is the absolute value of the instantaneous velocity, $U(t) = U_0 + \hat{\imath} \cdot v(t)$.

Under the assumption that the kinematic viscosity and the thermal diffusivity are determined only by the bulk temperature, Prandtl number can be taken as constant for given conditions. When constant density and viscosity are assumed, Eq. (13) can be diminished to

$$Nu \propto |U|^{0.8} \tag{15}$$

Equation (15) suggests that the heat transfer is affected by the magnitude of the flow velocity. The flow transverse velocity, at the center, can be written [according to Eq. (3)] as

$$v(t) = A_v \cdot U_0 \cdot \cos\{[(\pi \cdot c)/d]t\} \tag{16}$$

Because in the case considered here the oscillation is perpendicular to the main flow, the relative magnitude of the transverse flow velocity at the center will be

$$|U|/U_0 = |[U_0 + \hat{i} \cdot v(t)]/U_0| = \sqrt{1 + A_v^2 \cdot \cos^2\{[(\pi \cdot c)/d]t\}} \tag{17}$$

When Eqs. (15) and (17) are combined, the dependence between the instantaneous Nusselt number amplification A, representing heat transfer amplification, and the velocity oscillation can be achieved,

$$A(t) = Nu(t)/Nu_0 = [|U|/U_0]^{0.8}$$

$$= \left(1 + A_v^2 \cdot \cos^2\{[(\pi \cdot c)/d]t\}\right)^{0.4} \tag{18}$$

Because the oscillation is of high frequency, and thermal inertia is considered, one may say that the time-averaged heat transfer amplification would be a more appropriate parameter. To obtain the time-averaged heat transfer amplification as a function of the velocity oscillation, Eq. (18) can be integrated over one oscillation cycle, or more. The correlation arrived at is

$$\bar{A} = \frac{\overline{Nu}}{Nu_0} = \frac{1}{\tau}\int_0^{\tau}\left[1 + A_v^2 \cdot \cos^2\left(\frac{\pi \cdot c}{d}t\right)\right]^{0.4} \mathrm{d}t \tag{19}$$

As a result, for a given pressure oscillation amplitude, the time-averaged heat transfer amplification can be calculated for a certain combustion chamber, using Eqs. (12) and (19). The results should not be sensitive to phase lag because time-averaged effect is considered.

For demonstration, this process has been theoretically applied to a liquid hydrogen/liquid oxygen combustion chamber of a 5.4 area ratio (between the combustor port and the nozzle throat), giving the results shown in Fig. 4. These results show that, for pressure amplitudes of up to 50% of the mean pressure, the heat transfer amplification can be as high as twofold and even more. As expected, these results, for transverse oscillations, are slightly higher than the results obtained by Bogdanoff[4] and by Hanby,[5] for longitudinal oscillations, at similar velocity oscillation amplitudes. As mentioned before, high oscillation amplitude (such as 50% of the mean pressure) may imply nonlinear wave behavior.

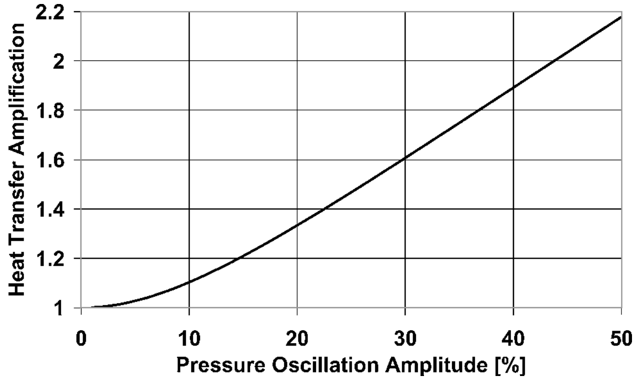

Fig. 4 Heat transfer amplification, LH_2/LO_2 combustion chamber.

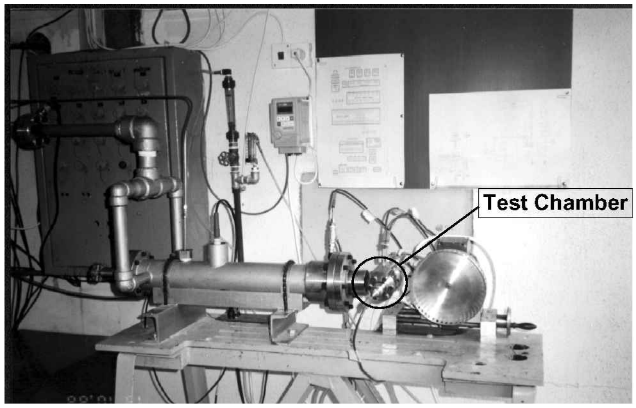

Fig. 5 Experimental apparatus.

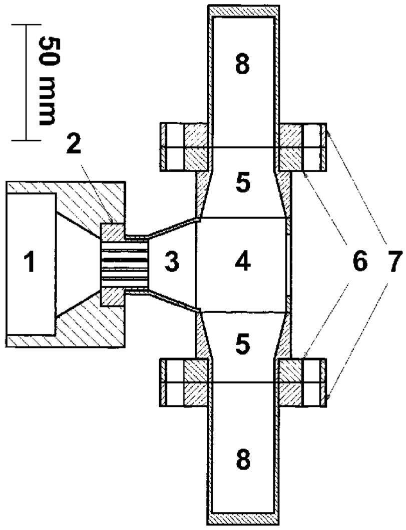

Fig. 6 Test chamber assembly: 1) flow adapter, 2) flow straightener, 3) flow adapter, 4) test chamber 5) flow adapter, 6) chamber flanges, 7) tube flanges, and 8) resonance tubes.

Experimental Apparatus

An experimental facility that simulates the flow conditions close to the wall in a liquid-propellant rocket engine has been designed and

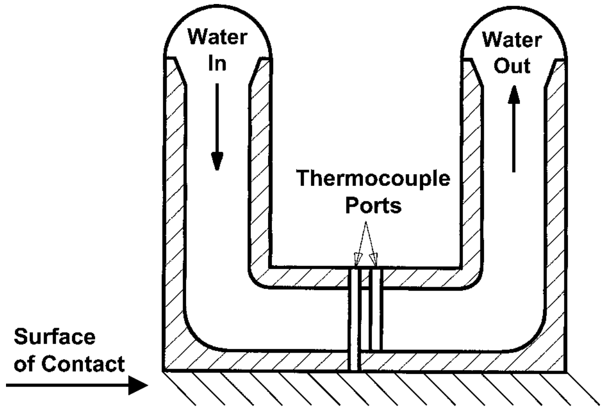

Fig. 7 Heat flux sensor.

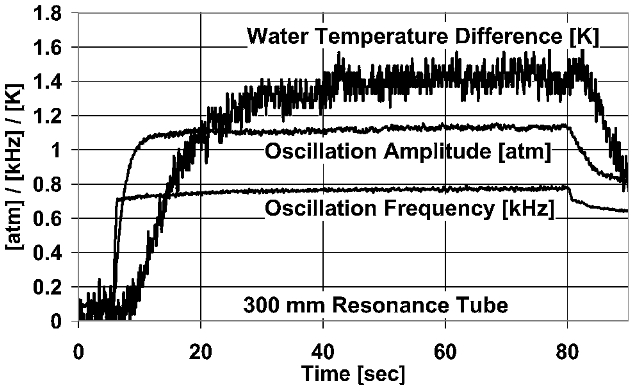

Fig. 8 Time traces.

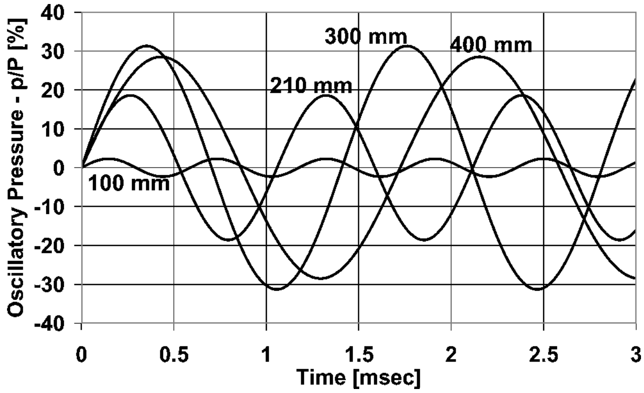

Fig. 9 Traces of pressure oscillations for different resonance tube lengths.

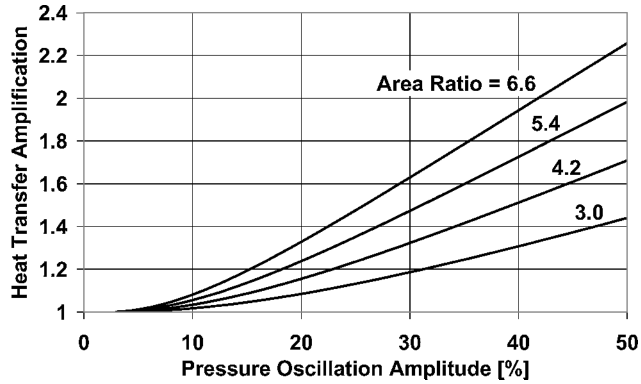

Fig. 10 Calculated wall heat transfer amplification for methane/air combustor.

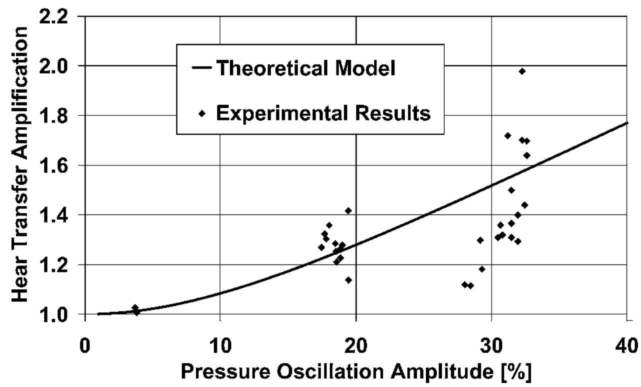

Fig. 11 Experimental and theoretical results of wall heat transfer amplification.

constructed (Fig. 5). The apparatus consists of a vitiated air heater, a rectangular test chamber (Fig. 6) with changeable resonance tubes perpendicular to the direction of the flow, and an excitation system, in the form of a rotating slotted wheel that blocks and opens the nozzle exit periodically at the desired frequency. Little use has been made of the excitation system, due to the spontaneous excitation of the first mode of oscillation.

Because of the relatively high temperature of the test chamber walls, a special heat flux sensor (HFS) was designed and built. The sensor, shown in Fig. 7, consisted of a stainless steel U-shaped water duct, with thermocouples at the water entrance and exit, at the inner wall of the duct, and at the surface of contact between the duct and the test chamber (wall 4 in Fig. 6).

Calculating the heat transfer amplification from the HFS measurements was done by two methods, one based on the heat capacitance of water and the other on wall conductivity. Under the assumptions of constant gas and water properties, during a specific set of tests, the correlation for the water heat capacitance method is

$$A = \frac{(T_{\text{water out}} - T_{\text{water in}})}{(T_{\text{water out}} - T_{\text{water in}})_0} \cdot \frac{(T_b - T_{\text{in wall}})_0}{(T_b - T_{\text{in wall}})} \tag{20}$$

and that for the wall conductivity method is

$$A = \frac{(T_{\text{mid wall}} - T_{\text{water}})}{(T_{\text{mid wall}} - T_{\text{water}})_0} \cdot \frac{(T_b - T_{\text{in wall}})_0}{(T_b - T_{\text{in wall}})} \tag{21}$$

Comparison of the values resulting from these two methods shows a difference of less than 3% for all of the experiments done, verifying their trustworthiness.

The experiments lasted 60–90 s each, allowing the system to attain its designated conditions. A time trace example is given in Fig. 8.

Four sets of resonance tubes were used, providing three high-amplitude oscillations and one low-amplitude oscillation. The pressure wave at the end of the resonance tube is shown in Fig. 9.

Results

The theoretical model was used to calculate heat transfer amplification for the case of methane (CH_4) and air combustion, applied in the experimental apparatus, showing the effect of the area ratio on the main flow heat transfer and, thereforth, on its amplification due to the oscillations (Fig. 10).

During this work, 33 experiments were performed in eight sets, differing in the conditions of the flow (chamber bulk temperature and pressure, as well as oscillation frequency and amplitude). Pressure oscillations obtained ranged between 3 and 33% of the mean pressure, where most of the oscillations concentrated around either 20 or 30%. The experimental results (Table 1) show good correspondence to the theoretical model of heat transfer amplification developed in this research (Fig. 11). In most cases, the scattering of the time-averaged amplification values, obtained from the experiments, has been within ±10% for medium oscillation amplitudes and ±15% for

Table 1 Summary of experimental results

Pressure oscillation amplitude, %	Heat transfer amplification	Oscillation frequency, Hz	Chamber bulk temperature, K
19.46	1.14	1084	516
18.53	1.25	1173	604
19.45	1.42	1231	665
19.00	1.28	1070	503
18.03	1.36	1156	586
18.58	1.21	1214	647
18.48	1.28	1168	599
18.90	1.26	1263	700
18.86	1.23	1263	700
18.78	1.26	1263	700
32.45	1.44	859	661
32.26	1.70	860	662
32.60	1.70	858	660
32.26	1.98	857	659
32.60	1.64	860	662
28.03	1.12	548	478
31.21	1.72	731	478
31.47	1.50	731	479
3.71	1.03	1682	516
3.85	1.01	1718	517
31.48	1.37	760	517
31.47	1.31	759	516
31.96	1.40	566	510
31.95	1.29	567	512
30.48	1.31	807	583
30.81	1.32	809	587
30.67	1.36	811	589
28.46	1.12	609	591
29.29	1.18	611	595
29.17	1.30	612	596
17.46	1.27	1163	594
17.69	1.32	1173	604
17.80	1.30	1177	609

high oscillation amplitudes. The dispersion of results, mainly at high oscillation amplitudes, can be related to several causes: heat losses to the surroundings and asymmetry and nonlinearity of the wave. Heat losses depend on the gas and ambient temperatures as well as on the exposed area, which changes according to the length of the resonance tubes. Asymmetry of the waveform may result from the different pressure gauges, mean and transient, which are mounted at opposite ends of the resonance tubes. Nonlinear effects, typical to high amplitude oscillations, are highly sensitive to variations in boundary and operating conditions.

Summary

The goal of this research was to determine the magnitude of the heat transfer amplification due to transverse mode of combustion instability. When the essential physical phenomena were accounted for, a simple, first-order evaluation model has been developed. A theoretical examination of the model sensitivity to bulk temperature resulted in a less than 2% change for a temperature range of 2000–3500 K. A special test facility was designed and built to generate experimental data and to examine the theoretical model.

The experimental results have demonstrated that the trend and approximate magnitude of the heat transfer amplification factor can be predicted using the proposed model.

The work presented here is primary. Further work is needed to better our understanding of the effect of tangential oscillation on heat transfer to the wall of a combustor.

Acknowledgments

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